

19. Prove that u and v be two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at a interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c . Then the function $f = u + iv$ has a derivative at c . Moreover $f'(c) = D_1 u(c) + iD_2 u(c)$.

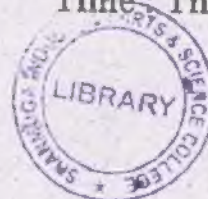
20. A quadric surface with center at the origin has the equation $Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$. Find the lengths of its semi-axes.

NOVEMBER/DECEMBER 2023

GMA22 — REAL ANALYSIS-II

Time : Three hours

Maximum : 75 marks



SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Lebesgue Integral.
2. Define Monotonic Sequences.
3. Define Riemann-integrable.
4. Define Measurable Functions.
5. Define Orthogonal Systems.
6. Define Dirichlet Integrals.
7. Directional Derivative.
8. Write down Taylor's formula.
9. Prove that $u + iv$ is a complex-valued function with a derivative at a point z in C in $f(z) = |f'(z)|^2$.
10. State Inverse Function theorem.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that $f \in U(I)$ and $g \in U(I)$, and if $f(x) = g(x)$ almost everywhere on I , Then $\int f = \int g$.

Or

- (b) Prove that $f \in U(I)$ and $\{s_n\}$ and $\{t_m\}$ be two sequences generating f . Then $\lim_{n \rightarrow \infty} \int s_n = \lim_{m \rightarrow \infty} \int t_m$.
12. (a) Prove that $f \in M(I)$ and if $|f(x)| \leq g(x)$ almost everywhere on I for some nonnegative g in $L(I)$, then $f \in L(I)$.

Or

- (b) Prove that $f \in L(I)$ and if f is bounded almost everywhere on I , then $f^2 \in L(I)$.
13. (a) Prove that g is of bounded variation on $[0, \delta]$,
Then $\lim_{\alpha \rightarrow +\infty} \frac{2}{\pi} \int_0^\delta g(t) \frac{\sin \alpha t}{t} dt$ is equal to $g(0+)$.

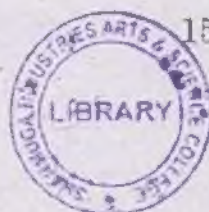
Or

- (b) Show that $x = \pi - 2 \sum_{n=1}^{\infty} \frac{\sin nx}{n}$ if $0 < x < 2\pi$.

14. (a) Prove that S be an open connected subset of R^n , and let $f : S \rightarrow R^m$ be differentiable at each point at each point of S . If $f'(c) = 0$ for each c in S , Then f is constant on S .

Or

- (b) State and Prove that Taylor's theorem.



15. (a) Prove that A be an open subset of R^n and $f : A \rightarrow R^n$ is continuous and has finite partial derivative $D_i f_i$ on A . if f is one-to-one on A and if $J_f(X) \neq 0$ for each x in A , then $f(A)$ is open.

Or

- (b) State and prove Implicit function theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and Prove Lebesgue dominated convergence theorem.
17. State and Prove Riesz- Fischer theorem.
18. State and Prove Weierstrass approximation theorem.